Plasma conditions for non-Maxwellian electron distributions in high current discharges and laser-produced plasmas

K. G. Whitney and P. E. Pulsifer
Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375

(Received 8 September 1992)

Results from the standard quasilinear theory of ion-acoustic and Langmuir plasma microturbulence are incorporated into the kinetic theory of the electron distribution function. The theory is then applied to high current discharges and laser-produced plasmas, where either the current flow or the nonlinear laser-light absorption acts, respectively, as the energy source for the microturbulence. More specifically, the theory is applied to a selenium plasma, whose charge state is determined under conditions of collisional-radiative equilibrium, and plasma conditions are found under which microturbulence strongly influences the electron kinetics. In selenium, we show that this influence extends over a wide range of plasma conditions. For ion-acoustic turbulence, a criterion is derived, analogous to one previously obtained for laser heated plasmas, that predicts when Ohmic heating dominates over electron-electron collisions. This dominance leads to the generation of electron distributions with reduced high-energy tails relative to a Maxwellian distribution of the same temperature. Ion-acoustic turbulence lowers the current requirements needed to generate these distributions. When the laser heating criterion is rederived with ion-acoustic turbulence included in the theory, a similar reduction in the laser intensity needed to produce non-Maxwellian distributions is found. Thus we show that ion-acoustic turbulence uniformly (i.e., by the same numerical factor) reduces the electrical and heat conductivities, as well as the current (squared) and laser intensity levels needed to drive the plasma into non-Maxwellian states. These effects are large over a wide range of plasma temperatures and densities such as found in Z-pinch, laser-produced, or opening switch plasmas. The reduction in laser intensity thresholds for generating non-Maxwellian states is a result of microturbulence enhanced laser absorption, which can be observed at any laser intensity. Thus, if microturbulence is the cause of the heat flux inhibition, then the magnitude of the inhibition might be directly observed through the use of probe-laser beams. Finally, we calculate the laser intensities and the current densities needed to change the electron distributions in the presence of ion-acoustic turbulence. These calculations are carried out by assuming that the electrostatic fluctuations scale as the square root of the electron plasma parameter in a weakly microturbulent selenium plasma. The laser intensities calculated in this example are found to be only slightly in excess of those used in recently conducted experiments.

PACS number(s): 52.25.Fi, 52.35.Qz

I. INTRODUCTION

Z-pinch plasmas are an important laboratory source of kilovolt x rays [1]. There are several possible energy sources for these x rays, whose contributions vary depending on the experiment [2]. For large aspect array implosions, i.e., ones initiated at radii larger than 1 cm, a large amount of kinetic energy can be generated during the acceleration of the array toward the axis [3]. In single-wire Z-pinch experiments or for low-aspect array implosions, however, the $\mathbf{j}\times\mathbf{B}$ current forces do most of their work directly on the plasma. For example, when the array wires are initially close to the axis, the wire explosion drives plasma inward, leading to the early generation of back pressure and to the performance of mostly pdV work, i.e., work done against this plasma pressure. A potential important third source of energy input, Ohmic heating, is also present in all experiments, although its magnitude is uncertain. Recent analyses of Z-pinch experiments [3,4], however, indicate that Ohmic heating may be competitive with the $\mathbf{j}\times\mathbf{B}$ energy inputs.

The reason for the difficulty in determining the magnitude of Ohmic heating is that Spitzer resistivity, which is calculated under the assumption of a nonturbulent, collisional plasma, is at least one order of magnitude smaller than is needed to explain experimental energy conservation [4]. One might conclude, therefore, that electrical resistivity in a Z pinch should be calculated using plasma turbulence theory. However, there are many historical difficulties to this task. For example, if one assumes that plasma microturbulence determines the resistivity, one must calculate the growth rate dynamics of different instabilities to determine how quickly they onset and saturate and at what levels they saturate. If the source of the additional resistivity is hydromagnetic turbulence, the calculational problem is even more difficult.

In laser-produced plasmas, there are similar problems associated with the absorption of laser light in the plasma. In this case, the absorption of electromagnetic energy by inverse bremsstrahlung can be strongly perturbed by the competing processes of stimulated Raman or stimulated Brillouin scattering. These scatterings can generate a strong microturbulence through the nonlinear generation of plasma waves and ion-acoustic waves [5]. As in the case of discharge plasmas, it is difficult to calculate the absorptivity of laser-generated plasmas in the presence of this microturbulence unless the dynamical growth and saturation of the microturbulence energy spectrum can be calculated throughout the plasma.

Microturbulence also affects the heat transport and viscosity properties of a collisional plasma as well as the shape of the isotropic electron distribution function [6,7]. These microturbulence properties will manifest themselves in a variety of ways in Z-pinch or laser-produced plasmas, and the full combination of their effects should be used to diagnose its presence. Moreover, microturbulence may need to be factored into any analysis of laser-produced plasma dynamics before other effects such as nonlocal heat transport can be invoked [8] to fully explain, for example, the inhibited heat flow known to be present in these plasmas [9].

In this paper, we assume that a fully developed microturbulence has been generated in either a high current discharge or a laser-produced plasma. Depending on the charge state of these plasmas, the microturbulence will change their electromagnetic absorption as well as their transport properties. Therefore, in order to demonstrate the size and extent of these changes, one must calculate this charge state for a particular plasma. We will specialize to selenium and calculate its charge state in collisional-radiative equilibrium (CRE), utilizing the atomic model described in Ref. [10]. In laser-produced plasmas, both Langmuir (or plasma wave) and ion-acoustic turbulence can be generated as a result of the laser-plasma interaction. However, in discharge plasmas, it is more common that the discharge current excites (primarily) ion-acoustic waves. Thus, for example, ion-acoustic turbulence was investigated earlier [11] and found to play an important role in the dynamics of theta pinches. More recently, it was found to have an important influence on the state of dense Z pinches in Pease-Braginskii equilibrium [12]. Furthermore, we have carried out a postanalysis of a nonturbulent magnetohydrodynamics calculation of a high-current (>10 MA) Z-pinch implosion and found that plasma conditions were established locally in the pinch during run in that produced current drift speeds in excess of the ion-acoustic sound speed c_s—a condition that is needed to trigger ion-acoustic instabilities. It was found that growth rates of this instability were sufficiently large to allow it to saturate during the current rise time of the pinch.

Because we are comparing the effects of microturbulence in both laser-produced and high-current-discharge plasmas, we initially investigate the coupling of both Langmuir and ion-acoustic turbulence to the electron kinetic equation. The focus in the remainder of the paper, however, will be on ion-acoustic microturbulence. We then seek to determine the plasma conditions and ion-acoustic fluctuation levels needed to significantly enhance a weakly turbulent selenium plasma's electrical resistivity or laser absorptivity given the CRE charge state of the plasma as a function of electron temperature and ion density. Microturbulence also promotes the production of self-similar non-Maxwellian electron velocity distributions of the kind described by Dum [6] and Langdon [13]. Langdon's criterion for non-Maxwellian distribution production is modified by the presence of a fully developed microturbulence, and this criterion is here generalized to high-current, microturbulent discharges. The laser intensities and current strengths that are needed to drive selenium into non-Maxwellian states can be determined from these criteria. These results may have special significance to x-ray laser research. Neonlike selenium is a well-studied x-ray laser medium [14], and a number of discrepancies regarding its observed, versus its calculated, ionization behavior remain to be explained [15]. Some of this discrepancy might be explained by the presence of microturbulence and its promotion of non-Maxwellian ionization behavior.

This paper is structured as follows. Section II contains a new derivation, more general than the one presented by Dum [7], of the way microturbulence can be included in the kinetic theory of a collisional plasma. Separate contributions of microturbulence to the different terms of the expansion of the distribution function in spherical harmonics are derived. For ion-acoustic turbulence, these contributions are, to good approximation, identical in form to the ones made by electron-ion collisions. In Sec. III, two criteria are derived for the strengths of the current density in an electrical discharge and the laser intensity in a laser-produced plasma that are needed to drive the electrons into a non-Maxwellian state due to rapid heating. They generalize the criterion derived previously [13]. Using a phenomenological model for the fluctuation level scaling achievable in weakly turbulent plasmas [16], we then calculate the selenium plasma conditions under which microturbulence would be expected to strongly affect the plasma's electrical and heat conductivities. We also calculate the required current densities and laser intensities needed to produce non-Maxwellian electron distributions as a function of selenium plasma conditions. Finally, a discussion of the implications of this work for experimentally diagnosing plasma microturbulence is given in Sec. IV.

II. MICROTURBULENCE ANALYSIS

We begin our analysis by writing the electron distribution function f_e as the sum of an average (in space and time) $\langle f \rangle$ and of a rapidly fluctuating distribution δf: $f_e = \langle f \rangle + \delta f$ [17]. We work in the coordinate system of the fluid flow, Fourier transforming the spatial coordinates from $\mathbf{x}$ to $\mathbf{k}$, and utilizing standard quasilinear theory to solve for δf by ignoring particle accelerations and plasma fluid flow [17]:

$$\delta f(\mathbf{k},\mathbf{v}) = \frac{ie}{m} \frac{\delta \mathbf{E}(\mathbf{k})}{\omega(\mathbf{k}) - \mathbf{k}\cdot\mathbf{v} + i\gamma(\mathbf{k})} \cdot \nabla_v \langle f \rangle \;, \tag{1}$$

where e is the electronic charge, a positive number, and m is the electron mass. The time dependence of the electromagnetic fluctuations is given by $\exp\{i[\omega(\mathbf{k}) + i\gamma(\mathbf{k})]t\}$. This expression for δf must be substituted into the equation for $\langle f \rangle$, which contains a collision term $C(\langle f \rangle)$ that is the sum of electron-

electron, electron-ion, and anomalous (electron-microturbulence) interactions:

$$C = C^{ee} + C^{ei} + C^{\rm an} , \tag{2}$$

where

$$C^{\rm an} \equiv (\partial_t \langle f \rangle)_{\rm an} = \frac{e}{m} \langle \delta \mathbf{E}(\mathbf{r},t) \cdot \nabla_v \delta f(\mathbf{r},\mathbf{v},t) \rangle . \tag{3}$$

Note, when we specialize to ion-acoustic microturbulence, $C^{\rm an}$ will be replaced by C^{ia}. When Eq. (1) is substituted into Eq. (3), a result that can be written in terms of a microturbulence diffusion tensor $\mathcal{D}_{ij}$ is obtained:

$$C^{\rm an} = \frac{\partial}{\partial v_i} \left[\mathcal{D}_{ij} \frac{\partial \langle f \rangle}{\partial v_j} \right] , \tag{4}$$

where

$$\mathcal{D}_{ij} = \frac{8\pi e^2}{m^2} \int d^3k \, \mathcal{W}(\mathbf{k}) \hat{\mathbf{k}}_i \hat{\mathbf{k}}_j \frac{\gamma(\mathbf{k})}{[\omega(\mathbf{k}) - \mathbf{k}\cdot\mathbf{v}]^2 + \gamma(\mathbf{k})^2} \tag{5}$$

$$= \mathcal{D}_0(v^2)\delta_{ij} + \mathcal{D}_1(v^2) \frac{v_i v_j}{v^2} . \tag{6}$$

$\mathcal{W}(\mathbf{k})$ is the energy spectrum of the microturbulence fluctuations, i.e., $\langle |\delta E(\mathbf{r},t)|^2/8\pi \rangle = \int d^3k \, \mathcal{W}(\mathbf{k})$, and $\hat{\mathbf{k}}_i \equiv \mathbf{k}_i / k$. We are interested in evaluating $\mathcal{D}_{ij}$ for those physical situations in which the turbulence is fully developed and the plasma is marginally stable [18]. In these situations, the limit $\gamma \rightarrow 0$ can be taken so that $\gamma/[(\omega - \mathbf{k}\cdot\mathbf{v})^2 + \gamma^2] \rightarrow \pi\delta(\omega - \mathbf{k}\cdot\mathbf{v})$. We further simplify the problem by assuming that the microturbulence is isotropic. (The more complete theory of nonisotropic microturbulence is reviewed by Bychenkov, Silin, and Uryupin [19].)

Langmuir and ion-acoustic microturbulence are two cases of particular interest. The dispersion relation for plasma waves (Langmuir microturbulence) is $\omega(\mathbf{k}) = \pm(\omega_e^2 + \frac{3}{2} v_{\rm th}^2 k^2)^{1/2}$, where $v_{\rm th} \equiv \sqrt{2k_B T_e/m}$ is the average electron thermal velocity and $\omega_e \equiv (4\pi n_e e^2/m)^{1/2}$ is the electron plasma frequency. For ion-acoustic waves, $\omega(\mathbf{k}) = \pm[k^2 c_s^2/(1 + k^2 \lambda_D^2)]^{1/2} \cong \pm k c_s$, where $c_s \cong \sqrt{\bar{Z} k_B T_e / m_i}$, and $\lambda_D = v_{\rm th}/(\sqrt{2}\omega_e)$ is the electron Debye length. $\bar{Z}$ is the average charge state of an ion, and m_i is the ion mass. The diffusion tensor can now be evaluated in these two cases by letting the spherical coordinate unit vectors $\hat{\mathbf{u}}_\theta, \hat{\mathbf{u}}_\phi$, and $\hat{\mathbf{u}}_v$ in velocity space define the directions $\hat{\mathbf{k}}_x, \hat{\mathbf{k}}_y$, and $\hat{\mathbf{k}}_z$ in k space, respectively. The k-space integration in Eq. (5) can be carried out in the spherical k-space coordinates (k, θ', ϕ'), where $\mathbf{k}\cdot\mathbf{v} = kv \cos\theta' \equiv kv\,\mu$. Using the reality condition $\omega(-\mathbf{k}) = -\omega(\mathbf{k})$, we rewrite Eq. (5) as

$$\mathcal{D}_{ij} = \frac{8\pi^2 e^2}{m^2} \int_0^\infty k^2 dk \, \mathcal{W}(k) \int_0^{2\pi} d\phi' \left\{ \int_{-1}^0 d\mu \, \hat{\mathbf{k}}_i \hat{\mathbf{k}}_j \delta(kv\,\mu + \omega(k)) + \int_0^1 d\mu \, \hat{\mathbf{k}}_i \hat{\mathbf{k}}_j \delta(kv\,\mu - \omega(k)) \right\} , \tag{7}$$

where $\omega(k) = (\omega_e^2 + \frac{3}{2} v_{\rm th}^2 k^2)^{1/2}$ for plasma waves and $\omega(k) = k c_s$ for ion-acoustic waves. The integrations produce the following diagonal diadic form:

$$\mathcal{D} = \mathcal{D}_0(\hat{\mathbf{u}}_\theta \hat{\mathbf{u}}_\theta + \hat{\mathbf{u}}_\phi \hat{\mathbf{u}}_\phi) + (\mathcal{D}_0 + \mathcal{D}_1)\hat{\mathbf{u}}_v \hat{\mathbf{u}}_v . \tag{8}$$

For Langmuir turbulence,

$$\mathcal{D}_0 = \frac{4\pi^2 e^2}{m^2 v^3} \{ (v^2 - \tfrac{3}{2} v_{\rm th}^2)\Omega_1 - \omega_e^2 \Omega_{-1} \} \eta_+(v - \sqrt{3/2}\, v_{\rm th}) , \tag{9}$$

$$\mathcal{D}_1 = -\frac{4\pi^2 e^2}{m^2 v^3} \{ (v^2 - \tfrac{9}{2} v_{\rm th}^2)\Omega_1 - 3\omega_e^2 \Omega_{-1} \} \eta_+(v - \sqrt{3/2}\, v_{\rm th}) , \tag{10}$$

while, for ion-acoustic turbulence,

$$\mathcal{D}_0 = \frac{4\pi^2 e^2 \Omega_1}{m^2 v} \left[1 - \frac{c_s^2}{v^2} \right] \eta_+(v - c_s) , \tag{11}$$

$$\mathcal{D}_1 = -\frac{4\pi^2 e^2 \Omega_1}{m^2 v} \left[1 - \frac{3c_s^2}{v^2} \right] \eta_+(v - c_s) . \tag{12}$$

In these expressions,

$$\Omega_1 \equiv 4\pi \int_{k_{\rm min}}^\infty dk \, k \mathcal{W}(k) , \tag{13}$$

$$\Omega_{-1} \equiv 4\pi \int_{k_{\rm min}}^\infty \frac{dk}{k} \mathcal{W}(k) . \tag{14}$$

$k_{\rm min} \equiv \omega_e/(v^2 - \frac{3}{2} v_{\rm th}^2)^{1/2}$ for Langmuir turbulence, and $k_{\rm min} = 0$ for ion-acoustic turbulence. $\eta_+(x)$ is the Heaviside function, which equals one when $x > 0$ and is zero otherwise. The spectrum integrations Ω_1 and Ω_{-1} define the strength of the Langmuir and ion-acoustic microturbulences. Note that Langmuir turbulence directly affects only the high energy tail of the electron distribution function, since $\mathcal{D}_{ij} = 0$ unless $v > \sqrt{1.5}\, v_{\rm th}$. However, ion-acoustic turbulence affects essentially the entire distribution function, because $c_s \ll v_{\rm th}$, and one can take

$$\mathcal{D}_0 \cong -\mathcal{D}_1 \cong \frac{4\pi^2 e^2 \Omega_1}{m^2 v} \tag{15}$$

to good approximation in order to describe these interactions.

We next make the usual approximate expansion of $\langle f \rangle$ in spherical harmonics [20] in the coordinate system moving with the fluid velocity $\mathbf{v}_f$:

$$\langle f \rangle \cong f_0 + \mathbf{f}_1 \cdot \frac{\mathbf{v}}{v} . \tag{16}$$

The collision terms are similarly expanded: $C \cong C_0 + \mathbf{C}_1 \cdot \mathbf{v}/v$. The equations of motion for f_0 and $\mathbf{f}_1$ are then (see Ref. [21], pp. 114 and 115)

$$(\partial_t+\mathbf{v}_f\cdot\nabla)f_0-(\nabla\cdot\mathbf{v}_f)\frac{v}{3}\frac{\partial f_0}{\partial v}+\frac{v}{3}\nabla\cdot\mathbf{f}_1+\frac{1}{3v^2}\frac{\partial}{\partial v}(v^2\mathbf{a}'\cdot\mathbf{f}_1)=C_0(f_0) \quad (17)$$

and

$$(\partial_t+\mathbf{v}_f\cdot\nabla)\mathbf{f}_1-(\nabla\mathbf{v}_f)\cdot\mathbf{f}_1-[\nabla\,\mathbf{v}_f+(\nabla\mathbf{v}_f)^T+\mathbf{I}_2\nabla\cdot\mathbf{v}_f]\cdot\frac{v^2}{5}\frac{\partial}{\partial v}\left[\frac{\mathbf{f}_1}{v}\right]-\omega_c\times\mathbf{f}_1-\mathbf{C}_1(\mathbf{f}_1)=-v\nabla f_0-\mathbf{a}'\frac{\partial f_0}{\partial v}\ , \quad (18)$$

where

$$\mathbf{a}'\cong-\frac{e}{m}\left[\mathbf{E}+\frac{\mathbf{v}_f}{c}\times\mathbf{B}\right]\equiv-\frac{e}{m}\mathbf{E}'\ . \quad (19)$$

$\mathbf{E}'$ is the effective electric field seen in the local frame of the moving fluid, and $\omega_c\equiv e\mathbf{B}/mc$. Both C_0 and $\mathbf{C}_1$ contain contributions from C^{an}, which we denote C_0^{an} and $\mathbf{C}_1^{\mathrm{an}}$, respectively. From Eqs. (4) and (6), one finds that

$$C_0^{AN}=\frac{1}{v^2}\partial_v[v^2(\mathcal{D}_0+\mathcal{D}_1)\partial_v f_0] \quad (20)$$

and

$$\mathbf{C}_1^{AN}=\frac{1}{v^2}\partial_v[v^2(\mathcal{D}_0+\mathcal{D}_1)\partial_v\mathbf{f}_1]-\frac{2}{v^2}\mathcal{D}_0\mathbf{f}_1\ . \quad (21)$$

(Similarly, by going one more term in the spherical harmonic expansion, one obtains $\mathbf{C}_2^{\mathrm{an}}=v^{-2}\partial_v[v^2(\mathcal{D}_0+\mathcal{D}_1)\partial_v\mathbf{f}_2]-6v^{-2}\mathcal{D}_0\mathbf{f}_2$.)

For ion-acoustic turbulence, Eq. (15) implies that $C_0^{ia}\cong 0$ and

$$\mathbf{C}_1^{ia}\cong-\frac{2}{v^2}\mathcal{D}_0\mathbf{f}_1 \quad (22)$$

$$\equiv-\nu^{ia}\left[\frac{v_{\mathrm{th}}}{v}\right]^3\mathbf{f}_1\ , \quad (23)$$

where the effective velocity-independent ion-acoustic collision frequency ν^{ia} is

$$\nu^{ia}=\frac{8\pi^2e^2\Omega_1}{m^2v_{\mathrm{th}}^3}\ . \quad (24)$$

Thus, like electron-ion scattering (and unlike electron–plasma-wave interactions), the interaction of electrons with ion-acoustic waves primarily affects the momentum transfer to or from the electrons. Because of the simplicity of the ion-acoustic collision terms, we will further simplify the following discussion by focusing on electron–ion-acoustic-wave interactions.

We also adopt the common procedure of ignoring electron-ion collisions in Eq. (17) ($C_0^{ei}\cong 0$) and electron-electron collisions in Eq. (18) ($\mathbf{C}_1^{ee}\cong\mathbf{0}$). The isotropic electron-electron collision term C_0^{ee} is given by the Fokker-Planck expression [21]. If the ions are Maxwellian, the electron-ion momentum transfer in the ion rest frame has the same form as the electron–ion-acoustic term [21]:

$$\mathbf{C}_1^{ei}=-\nu_{ei}\left[\frac{v_{\mathrm{th}}}{v}\right]^3\mathbf{f}_1\ , \quad (25)$$

where the velocity-independent electron-ion collision frequency is defined by

$$\nu_{ei}\equiv\frac{4\pi n_e\bar{Z}e^4\ln_{10}\Lambda_e}{m^2v_{\mathrm{th}}^3}\ , \quad (26)$$

and $\ln_{10}\Lambda_e$ is the Coulomb logarithm. Thus

$$C_0\cong C_0^{ee} \quad (27)$$

and

$$\mathbf{C}_1\cong\mathbf{C}_1^{ei}+\mathbf{C}_1^{ia}\equiv-\nu_1\left[\frac{v_{\mathrm{th}}}{v}\right]^3\mathbf{f}_1\ , \quad (28)$$

where

$$\nu_1=\nu_{ei}+\nu^{ia}\equiv(1+\beta)\nu_{ei}\ . \quad (29)$$

A similar expression for $\mathbf{C}_1$ was derived by Dum [7], who carried out a direct angle averaging of Eq. (5).

The effect of current and heat flow on the shape of the isotropic distribution function f_0 is determined by the two $\mathbf{f}_1$ terms in Eq. (17). The first term, involving $\nabla\cdot\mathbf{f}_1$, determines the effects of current and heat flow on the electron number and energy densities, respectively. The second term, involving $\mathbf{a}'$, represents the effect of Ohmic heating on the electrons. To determine $\mathbf{f}_1$, we look for a solution to Eq. (18). Generally, as a first approximation, the gradient and time derivative terms can be dropped, leaving one with the equation,

$$-\omega_c\times\mathbf{f}_1+(1+\beta)\nu_{ei}\left[\frac{v_{\mathrm{th}}}{v}\right]^3\mathbf{f}_1=-v\nabla f_0-\mathbf{a}'\frac{\partial f_0}{\partial v}\ . \quad (30)$$

This equation may be easily solved for $\mathbf{f}_1$ in terms of $f_0(v)$. If a Maxwellian distribution is used for f_0, simple expressions can then be found for the electric current $\mathbf{j}_e$ and heat flux $\mathbf{q}_e$ from which the Lorentz conductivity σ_L and thermal conductivity κ_e can be obtained.

For our evaluation of $\mathbf{j}_e$, we neglect the modifications of σ_L (approximately equal to a factor of 2 or 3) that are produced by the B field by setting $\omega_c=0$. Since this calculation is primarily applied to discharge plasmas, we further neglect the effect of pressure gradients, embodied in the ∇f_0 term, to drive plasma currents. A standard looking result for $\mathbf{j}_e$ is then obtained, but one that now includes the potentially substantial influence of ion-acoustic microturbulence:

$$\mathbf{j}_e=-\frac{4\pi}{3}e\int_0^\infty dv\,v^3\mathbf{f}_1(v) \quad (31)$$

$$=\frac{4\pi}{3}\frac{e\mathbf{a}'}{v_{\mathrm{th}}^3\nu_{ei}(1+\beta)}\int_0^\infty dv\,v^6\frac{\partial f_0}{\partial v}\equiv\sigma_L\mathbf{E}'\ , \quad (32)$$

where

$$\sigma_L = \left[\frac{32}{3\pi(1+\beta)} \right] \frac{n_e e^2 \tau_e}{m} , \tag{33}$$

and $\tau_e \equiv 3\sqrt{\pi}/(4\nu_{ei})$ is the electron-ion collision time [21]. [Spitzer's conductivity, which is calculated by the inclusion of electron-electron collisions in Eq. (30), would correct this result by roughly a factor of 2 or less.]

In order to determine the effect of ion-acoustic turbulence on the conductivities of a laser-produced plasma, one would take a complementary approach to the one used above. In the absence of current flow, the main conductivity effect would be the reduction of heat flow from the critical to the ablation surface. The steady-state acceleration is zero, and the source term for $\mathbf{f}_1$ comes from the ∇f_0 term in Eq. (30). In this case, the second moment of $\mathbf{f}_1$ must be computed:

$$\mathbf{q}_e \cong \frac{2\pi m}{3} \int_0^\infty dv\, v^5 \mathbf{f}_1(v) \tag{34}$$

$$\cong -\kappa_e \nabla(k_B T_e) , \tag{35}$$

and one finds that

$$\kappa_e = c_e^T \frac{n_e k_B T_e}{m} \frac{\tau_e}{1+\beta} , \tag{36}$$

where c_e^T is a constant. Thus both the electrical and the heat conductivity, which are proportional to the effective electron-ion collision time $\tau_e^{\text{eff}} \equiv \tau_e/(1+\beta)$, are reduced by ion-acoustic turbulence by the same factor $(1+\beta)$, i.e., a given level of microturbulence that produces an anomalously large amount of Ohmic heating will also inhibit heat flow by the same amount and vice versa.

To determine the size and the dynamics of these conductivity reductions in detail, one has the difficult task of calculating the turbulence spectrum $\mathcal{W}$. However, since only one moment Ω_1 of $\mathcal{W}$ is needed to determine the size of β for ion-acoustic turbulence, it is possible to estimate how strongly a given level of electrostatic microturbulence affects the electrical resistivity $\eta_L \equiv 1/\sigma_L$ and the heat conductivity κ_e by establishing a bound on Ω_1. We begin with the definition of β. From Eqs. (24) and (29),

$$\beta = \frac{3\pi}{2} \left[\frac{\omega_e}{v_{\text{th}}} \right] \left[\frac{\Omega_1}{E_{\text{th}}} \right] \frac{\omega_e}{\nu_{ei}} , \tag{37}$$

where E_{th} is the electron thermal energy density:

$$E_{\text{th}} = \tfrac{3}{2} n_e k_B T_e . \tag{38}$$

The size of Ω_1 can be estimated by factoring an average ion-acoustic wavelength $1/\langle k \rangle$ from its integrand; then, $\Omega_1 = f_E E_{\text{th}}/\langle k \rangle$, where f_E is the ratio of the electrostatic fluctuation energy to the thermal energy $(\langle |\delta E(\mathbf{r},t)|^2/8\pi \rangle / E_{\text{th}})$. Thus

$$\beta = \frac{3\pi}{2} \left[\frac{\omega_e/\langle k \rangle}{v_{\text{th}}} \right] \left[\frac{\omega_e}{\nu_{ei}} \right] f_E . \tag{39}$$

If, by virtue of the ion-acoustic dispersion relation, one assumes that $\langle k \rangle \le 1/\lambda_D$, then

$$\beta \ge \beta_{\min} \equiv \sqrt{2\pi}\, \omega_e \tau_e f_E . \tag{40}$$

For Z-pinch and laser-produced plasmas, the product $\omega_e \tau_e$ can range in value from >1 to $>10^4$; thus, for large values of this ratio, ion-acoustic energy fluctuation levels f_E of 1% can easily lead to a turbulence contribution to the electrical resistivity that is ten times larger than that due to two-particle collisions. This same turbulence level corresponds to a heat flux limiter of 0.1, a value often used to describe the dynamics of laser-produced plasmas [9].

III. CRITERIA FOR NON-MAXWELLIAN DISTRIBUTIONS

Investigating the nonturbulent laser heating of electrons, Langdon [13] derived a criterion that predicted the intensity at which inverse bremsstrahlung heating would dominate over electron-electron collisions. He found that, when this criterion was satisfied, the electron distribution approached a self-similar form whose high-energy tail was depleted relative to that of a Maxwellian distribution. Earlier, Dum [6] had found that, under strong ion-acoustic turbulence, the electron distribution tends towards the same self-similar form as discussed by Langdon. We derive a criterion, analogous to Langdon's, for the current density required in a high current, microturbulent discharge to produce non-Maxwellian distributions, and we reexamine Langdon's derivation to determine how his laser criterion is modified by a microturbulent plasma.

By substituting the general solution to Eq. (30) into Eq. (17) and by ignoring the $v\nabla f_0$ term in Eq. (30) and the gradient terms in Eq. (17), one obtains the equation $\partial_t f_0 = H(f_0) + C_0(f_0)$ in the same form as that used by Langdon to discuss the intense heating of high atomic number laser-produced plasmas. When magnetic fields are present to inhibit electron runaway, the Ohmic heating term $H(f_0)$ has the same form as the laser heating term:

$$H(f_0) = \frac{e^2 E'^2}{m^2} \frac{1}{3v^2} \frac{\partial}{\partial v} \left[v^2 \frac{\nu_1 (v_{\text{th}}/v)^3}{\omega_c^2 + \nu_1^2 (v_{\text{th}}/v)^6} \frac{\partial f_0}{\partial v} \right] . \tag{41}$$

The characteristic growth time τ_H^{Ohmic} for Ohmic heating that is analogous to Langdon's e-folding time for laser heating is

$$\tau_H^{\text{Ohmic}} \equiv \frac{E_{\text{th}}}{\mathbf{j}\cdot\mathbf{E}'} = \frac{\sigma_L E_{\text{th}}}{j_e^2} , \tag{42}$$

where $j_e^2/\sigma_L = 4\pi \int dv\, v^2 (m/2) v^2 H(f_0)$. $H(f_0)$ dominates $C_0(f_0)$ when the Ohmic (as with the laser) e-folding time for heating is smaller than the equilibration collision time τ_{ee} needed to establish a Maxwellian distribution: $\tau_{ee}/\tau_H^{\text{Ohmic}} \ge 1$. Since $j_e = n_e e v_d$, where v_d is the average electron drift speed in the local rest frame of the fluid, and since $\tau_{ee} = \bar{Z}\tau_e$, one can write this criterion as

$$\tau_{ee}/\tau_H^{\text{Ohmic}} = \frac{\pi}{8}(1+\beta)\bar{Z} \frac{v_d^2}{v_{\text{th}}^2} \ge 1 , \tag{43}$$

using Eq. (33). This inequality determines the drift speed or equivalently the current strength needed for Ohmic heating to generate significant non-Maxwellian isotropic electron distributions under different plasma conditions. The analogous formula of Langdon is $\overline{Z}v_0^2/v_{\rm th}^2 \geq \frac{3}{2}$, where v_0 is the peak oscillation velocity of the electrons in the high frequency (laser) electric field.

One can determine when the non-Maxwellian criterion [Eq. (43)] is satisfied in two different ways. On the one hand, by using the commonly employed idea that the electron drift speed v_d saturates at the ion sound speed [22] in a fully developed ion-acoustic microturbulence, one has that

$$\frac{v_d^2}{v_{\rm th}^2}=\frac{c_s^2}{v_{\rm th}^2}=\frac{\overline{Z}m}{2m_i}\ , \tag{44}$$

and the criterion $\tau_{ee}/\tau_H^{\rm Ohmic}\geq 1$ becomes

$$\frac{\pi}{16}(1+\beta)\overline{Z}^2\frac{m}{m_i}\geq 1\ . \tag{45}$$

From Ref. [3], $m_i \cong 1.58Z^{1.1}m_p$ for $Z\leq 36$, where m_p is the proton mass and Z is the atomic number of the plasma, and the non-Maxwellian condition becomes

$$(1+\beta)Z^{0.9}\frac{m}{m_p}\geq 8\ , \tag{46}$$

when the plasma is fully ionized. This condition is satisfied for $Z\leq 36$ only in a highly turbulent plasma: $\beta \sim 1000$.

On the other hand, in an inductively driven system like a Z pinch, it is not clear that turbulent resistivity can effectively limit $v_d \leq c_s$. An alternative and perhaps better way to view this criterion, therefore, is as a condition on the current density. One can then rewrite Eq. (43) as the non-Maxwellian criterion

$$j_e \geq j_{\rm crit}\ , \tag{47}$$

where

$$j_{\rm crit}=\frac{j_{\rm th}}{\sqrt{(\pi/8)(1+\beta)\overline{Z}}} \tag{48}$$

and $j_{\rm th}\equiv n_e e v_{\rm th}$. As an example, in a nearly fully ionized aluminum plasma at 1 keV, at an electron density of 10^{21} cm^{-3}, and with $\beta \cong 100$, one would need a current density in excess of 10^{10} A/cm^2 to drive the plasma into a strongly non-Maxwellian state, i.e., a current larger than 1 MA would have to flow in a skin depth of area 10^{-4} cm^2. Since $j_{\rm th}$ scales linearly with n_e, the current density requirement in plasma opening switches scales down accordingly.

Equations (46) and (47) show that microturbulence can play an important role in reducing the magnitude of the current density needed to generate non-Maxwellian electron distributions. If the laser criterion is rederived by including $\mathbf{C}_1^{ia}$ in the analysis, a similar reduction in the magnitude of the laser intensity needed to generate non-Maxwellians is found, i.e., the laser heating criterion for a laser of wavelength λ becomes

$$I/I_{\rm crit}=\tfrac{2}{3}(1+\beta)\overline{Z}\frac{v_0^2}{v_{\rm th}^2}\geq 1\ , \tag{49}$$

and, in this case,

$$I_{\rm crit}=\frac{3\pi m c^3 k_B T_e}{2\overline{Z}e^2(1+\beta)\lambda^2}\ \frac{\rm erg}{\rm sec\,cm^2}\ . \tag{50}$$

The reduction in $I_{\rm crit}$ by the factor $(1+\beta)$ corresponds to an increase in the laser absorption coefficient κ_L by the same amount since $(\mathbf{j}\cdot\mathbf{E})_{\rm laser}=\kappa_L I$.

Because the laser and Ohmic heating terms are identical in form in a strongly magnetized plasma, the electron distribution would approach the self-similar form $u^{-3}\exp(-v^5/5u^5)$, with $u^5 \sim t$, discussed by Dum [6] and Langdon [13], were Ohmic heating to completely dominate over electron collisions. This might happen by design in plasma opening switches, for example. For weaker current flows, the distribution function has transitional behavior of the kind calculated by Alaterre, Matte, and Lamoureux [23]. In all of these cases, the number of electrons in the tail of the distribution is reduced, leading to reductions in the rate at which the plasma is excited or ionized. Significant departures of f_0 from a Maxwellian distribution also change the calculation of $\mathbf{f}_1$, which, in turn, modifies the electrical and heat conductivities that are calculated due to plasma microturbulence alone (see Refs. [6] and [7]).

The magnitudes of the microturbulence corrections to σ_L, κ_e, $j_{\rm crit}$, and $I_{\rm crit}$ depend on plasma conditions and can be determined once the saturation level f_E is known. This level also depends on plasma conditions and on the strength of the energy source driving the turbulence. We investigate this dependence by arbitrarily fixing the value of f_E for one set of plasma conditions and by scaling f_E as described by Ichimaru [16]. In a weakly turbulent plasma, he argued that the turbulence spectrum scales as $g^{1/2}$, where g is the plasma parameter

$$g\equiv\frac{1}{n_e\lambda_D^3}\ , \tag{51}$$

which we take to be the electron plasma parameter. Note that in strongly turbulent plasmas, f_E would scale as g^0, i.e., it would be independent of g [16].

To illustrate how this scaling affects $\beta_{\rm min}$, $j_{\rm crit}$, and $I_{\rm crit}$, we specialize to selenium and calculate its ionization state $\overline{Z}$ using the atomic model described in Ref. [10] by assuming that the plasma is in CRE. Although laser-produced selenium plasmas have been well studied recently in x-ray laser experiments [14,15], these plasmas are also of interest in Z-pinch experiments. The $\overline{Z}$ values for selenium that are needed to calculate the quantities f_E, $\beta_{\rm min}$, $j_{\rm crit}$, and $I_{\rm crit}$ are displayed in Fig. 1 as a function of temperature and density. For the purpose of this numerical example, we took $f_E=0.5g^{1/2}$. This formula produced the values for f_E that are shown in Fig. 2 for the same plasma temperatures and densities as Fig. 1. Figure 2 is an illustration of the fact that, in weakly turbulent plasmas, as in quiescent plasmas, relatively higher levels of electrostatic energy fluctuations can be generat-

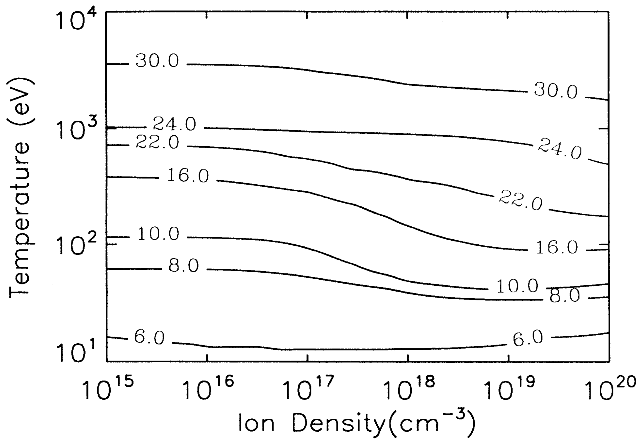

FIG. 1. Contours of the average charge state $\bar{Z}$ of selenium in CRE are given as a function of electron temperature and ion density.

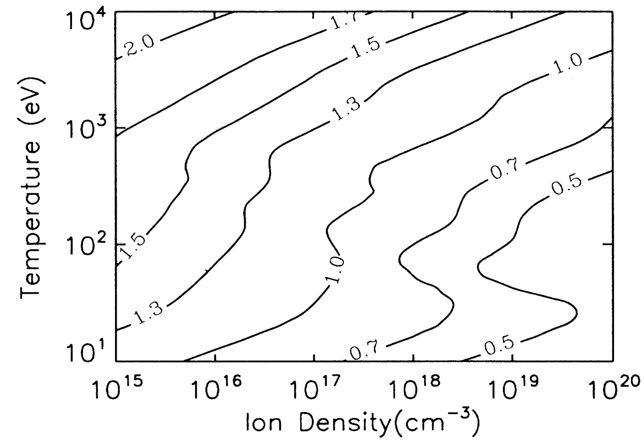

FIG. 3. Contours of $\log_{10}(\beta_{\min})$ are shown as a function of electron temperature and ion density. They were calculated using the values for $\bar{Z}$ and f_E shown in Figs. 1 and 2.

ed as the plasma temperature is lowered or as the plasma density is increased. These fluctuation levels produce the $\beta_{\min}$ enhancement factors [see Eq. (40)] shown in Fig. 3, which, in turn, produce the contours for j_{crit} and I_{crit} that are presented in Figs. 4 and 5, respectively. The I_{crit} values in Fig. 5 were calculated for frequency tripled neodymium ($\lambda = 0.35$ μm) laser radiation.

Figure 4 shows that fairly substantial current densities would be needed before self-similar non-Maxwellian distributions could be generated in discharge plasmas. The plasmas generated in present-day Z-pinch or plasma opening switch experiments have anomalously high electrical resistivities, which may indicate the presence of microturbulence. Present-day theoretical models do not predict the existence of current densities in Z pinches as large as those in Fig. 4; therefore it is unclear whether present-day pinch experiments exhibit any of the ion-acoustic non-Maxwellian behavior discussed above. However, if the higher-current machines (as in pulse power x-ray laser experiments [24]) are built that are needed to efficiently produce high-power kilovolt x rays from moderate Z elements [3], then the criterion for generating non-Maxwellian electron distributions may become increasingly easier to satisfy and to exceed. This observation may be true as well of scaled up x-ray laser experiments conducted in laser-produced plasmas. The I_{crit} values in Fig. 5 are close to those used in current experiments, so that as these experiments are scaled to higher photon energies or higher x-ray laser intensities, the likelihood for generating non-Maxwellian electron distributions under microturbulent plasma conditions may increase. Finally, we note that Fig. 2 can be scaled directly to calculate $\beta_{\min}$ in selenium for any f_E since $\beta_{\min}$ is directly proportional to f_E.

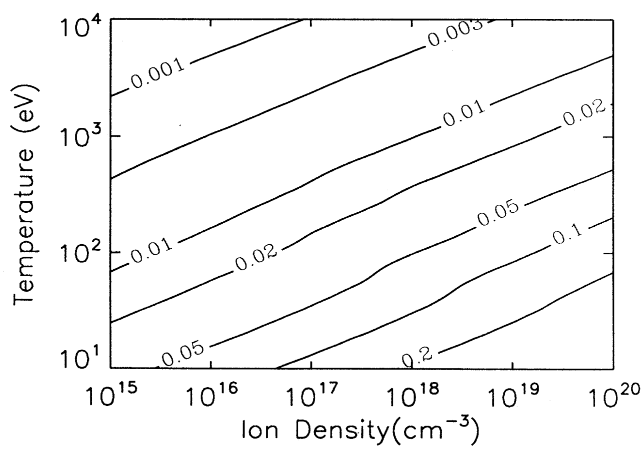

FIG. 2. Contours of $f_E = 0.5g^{1/2}$, where g is the electron plasma parameter, are shown as a function of electron temperature and ion density.

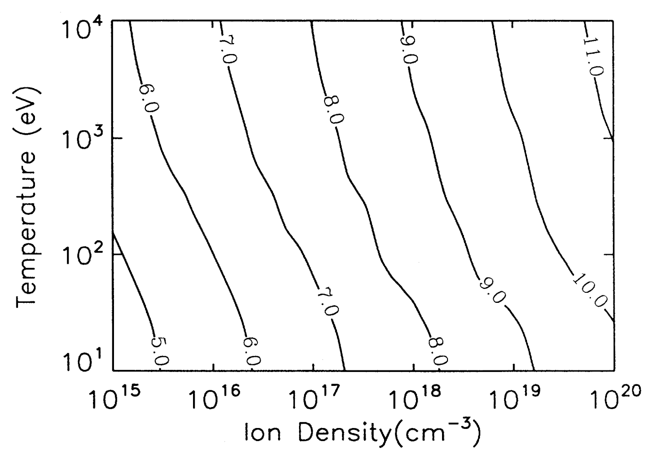

FIG. 4. Contours of $\log_{10}(j_{\text{crit}})$ are shown as a function of electron temperature and ion density using the $\bar{Z}$ and β values in Figs. 1 and 3.

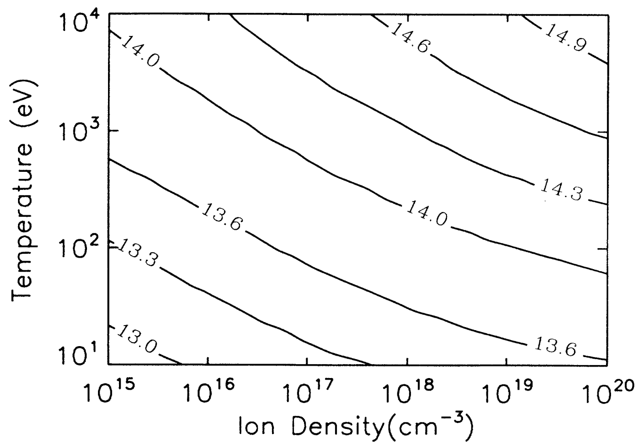

FIG. 5. Contours of $\log_{10}(I_{crit})$ are shown as a function of electron temperature and ion density using the $\bar{Z}$ and β values in Figs. 1 and 3 and a laser wavelength of 0.35 μm.

IV. SUMMARY AND CONCLUSIONS

Two major problems in plasma microturbulence theory are (1) to determine the dynamics of the electromagnetic energy fluctuations, inclusive of their initial growth rates, their saturation mechanisms, their propagation properties, and the dependence of these properties on inhomogeneous plasma conditions, and (2) to determine the effects of microturbulence on the electron kinetics and on the fluid dynamics of the plasma. While both of these problems are important, only the latter one was analyzed in this paper under the assumption that the microturbulence had been triggered and had grown to a saturated level that was some (small) fraction of the electron thermal energy. Under this assumption, we showed how microturbulence couples to the different terms in the spherical harmonic expansion of the electron kinetic equation for the cases of both Langmuir and ion-acoustic turbulence. By specializing to ion-acoustic turbulence, we found, in agreement with Ref. [7], that the essential effect of electron–ion-acoustic coupling is to increase the electron-ion collision frequency. In so doing, it alters four basic properties of the plasma (by essentially the same correction factor): electrical resistivity is increased, heat flow is reduced, and the critical current densities and laser intensities needed to generate non-Maxwellian distributions are reduced. The latter reduction occurs as a result of the increase in the laser absorption coefficient. This absorption change could lead to the use of probe-laser beams to determine the fluctuation levels of microturbulent plasmas experimentally.

Since microturbulence reduces the threshold for generating non-Maxwellian electron distributions, its presence would also manifest itself indirectly through this distribution's influence on the x-ray emissions generated in the plasma [25]. These emissions could, therefore, provide diagnostics of the microturbulence. Because the laser intensity threshold needed to produce non-Maxwellian distributions is significantly reduced by (ion-acoustic) microturbulence, our calculations suggest that these emission effects might first be seen in laser-produced plasma, x-ray laser experiments. More importantly, if non-Maxwellian distributions are generated in x-ray laser experiments conducted with high atomic number plasmas, then they could produce significant shifts in the ionization balance that would, in turn, significantly affect the design of these experiments.

ACKNOWLEDGMENT

The authors would like to thank M. R. Pauli for his careful reading of the manuscript and for his many helpful comments.

[1] *Dense Z-Pinches,* Proceedings of a Conference Concerning the Physics of Dense Linear *Z*-Pinches, Laguna Beach, CA, 1989, edited by Nino Pereira, Jack Davis, and Norman Rostoker, AIP Conf. Proc. No. 195 (AIP, New York, 1989), pp. 3–78.

[2] C. Deeney, T. Nash, R. R. Prasad, L. Warren, K. G. Whitney, J. W. Thornhill, and M. C. Coulter, Phys. Rev. A **44**, 6762 (1991).

[3] K. G. Whitney, J. W. Thornhill, J. P. Apruzese, and J. Davis, J. Appl. Phys. **67**, 1725 (1990); J. W. Thornhill, K. G. Whitney, and J. Davis, J. Quant. Spectrosc. Radiat. Transfer **44**, 251 (1990).

[4] M. Krishnan, C. Deeney, T. Nash, P. D. LePell, and K. Childers, in Ref. [1], p. 17.

[5] W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, Redwood City, CA, 1988).

[6] C. T. Dum, Phys. Fluids **21**, 945 (1978).

[7] C. T. Dum, Phys. Fluids **21**, 956 (1978).

[8] J. R. Albritton, E. A. Williams, and I. B. Bernstein, Phys. Rev. Lett. **57**, 1887 (1986); E. M. Epperlein and R. W. Short, Phys. Fluids B **3**, 3092 (1991).

[9] J. Delettrez, Can. J. Phys. **64**, 932 (1986); P. A. Holstein, J. Delettrez, S. Skupsky, and J. P. Matte, J. Appl. Phys. **60**, 2296 (1986).

[10] K. G. Whitney and M. C. Coulter, IEEE Trans. Plasma Sci. **16**, 552 (1988).

[11] P. C. Liewer and N. A. Krall, Phys. Fluids **16**, 1953 (1973).

[12] A. E. Robson, Phys. Rev. Lett. **63**, 2816 (1989).

[13] A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).

[14] C. J. Keane, N. M. Ceglio, B. J. MacGowan, D. L. Matthews, D. G. Nilson, J. E. Trebes, and D. A. Whelan, J. Phys. B **22**, 3343 (1989); R. A. London, M. D. Rosen, M. S. Maxon, D. C. Eder, and P. L. Hagelstein, *ibid.* **22**, 3363 (1989).

[15] M. Rosen, S. Maxon, B. LaFontaine, H. Baldis, J. Dunn, D. Villeneuve, G. Enright, and H. Pepin, Bull. Am. Phys. Soc. **36**, 2431 (1991).

[16] S. Ichimaru, *Basic Principles of Plasma Physics* (Benjamin, Reading, MA, 1973), Secs. 7.3, 11.3, and 11.4.
[17] R. C. Davidson and N. A. Krall, Nucl. Fusion **17**, 1313 (1977).
[18] W. M. Manheimer, J. Phys. (Paris) Colloq. **40**, C7-269 (1979).
[19] V. Yu. Bychenkov, V. P. Silin, and S. A. Uryupin, Phys. Rep. **164**, 119 (1988).
[20] T. W. Johnston, Phys. Rev. **120**, 1103 (1960).
[21] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, *The Particle Kinetics of Plasmas* (Addison-Wesley, Reading, MA, 1966).
[22] E. C. Field and B. D. Fried, Phys. Fluids **7**, 1937 (1964).
[23] P. Alaterre, J. P. Matte, and M. Lamoureux, Phys. Rev. A **34**, 1578 (1986).
[24] J. L. Porter, R. B. Spielman, M. K. Matzen, E. J. McGuire, L. E. Ruggles, M. F. Vargas, J. P. Apruzese, R. W. Clark, and J. Davis, Phys. Rev. Lett. **68**, 796 (1992).
[25] R. Bartiromo, F. Bombarda, and R. Giannella, Phys. Rev. A **32**, 531 (1985).